CHAPTER 11 (Odd)

1. Φ: CGS: **5×10^4 Maxwells**, English: **5×10^4 lines**
   B: CGS: **8 Gauss**, English: **51.616 lines/in.^{2}**

3. $B = \dfrac{\Phi}{A} = \dfrac{4 \times 10^{-4}\text{ Wb}}{0.01\text{ m}^2} = \mathbf{0.04\ T}$

5. $\Re = \dfrac{\mathscr{F}}{\Phi} = \dfrac{400\text{ At}}{4.2 \times 10^{-4}\text{ Wb}} = \mathbf{952.4 \times 10^3\ At/Wb}$

7. $6\text{ in.}\left[\dfrac{1\text{ m}}{39.37\text{ in.}}\right] = 0.1524\text{ m}$

   $H = \dfrac{\mathscr{F}}{l} = \dfrac{400\text{ At}}{0.1524\text{ m}} = \mathbf{2624.67\ At/m}$

9. $B = \dfrac{\Phi}{A} = \dfrac{10 \times 10^{-4}\text{ Wb}}{3 \times 10^{-3}\text{ m}^2} = 0.333\text{ T}$
   Fig. 11.23: $H \cong 800$ At/m
   $NI = Hl \Rightarrow I = Hl/N = (800\text{ At/m})(0.2\text{ m})/75\text{ t} = \mathbf{2.133\ A}$

11. a. $N_1I_1 + N_2I_2 = Hl$

    $B = \dfrac{\Phi}{A} = \dfrac{12 \times 10^{-4}\text{ Wb}}{12 \times 10^{-4}\text{ m}^2} = 1\text{ T}$
    Fig. 11.23: $H \cong 750$ At/m
    $N_1(2\text{ A}) + 30\text{ At} = (750\text{ At/m})(0.2\text{ m})$
    $N_1 = \mathbf{60\ t}$

    b. $\mu = \dfrac{B}{H} = \dfrac{1\text{ T}}{750\text{ At/m}} = \mathbf{13.34 \times 10^{-4}\ Wb/Am}$

13. $N_1I + N_2I = \underbrace{Hl}_{\text{cast steel}} + \underbrace{Hl}_{\text{cast iron}}$

    $(20\text{ t})I + (30\text{ t})I = $ "
    $(50\text{ t})I = $ "

    $B = \dfrac{\Phi}{A}$ with $0.25\,\cancel{\text{in.}}^2\left[\dfrac{1\text{ m}}{39.37\,\cancel{\text{in.}}}\right]\left[\dfrac{1\text{ m}}{39.37\,\cancel{\text{in.}}}\right] = 1.6 \times 10^{-4}\text{ m}^2$

    $B = \dfrac{0.8 \times 10^{-4}\text{ Wb}}{1.6 \times 10^{-4}\text{ m}^2} = 0.5\text{ T}$
    Fig. 11.24: $H_{\text{cast steel}} \cong 280$ At/m
    Fig. 11.23: $H_{\text{cast iron}} \cong 1500$ At/m

    $l_{\text{cast steel}} = 5.5\,\cancel{\text{in.}}\left[\dfrac{1\text{ m}}{39.37\,\cancel{\text{in.}}}\right] = 0.1397\text{ m}$

$$l_{\text{cast iron}} = 2.5\ \cancel{\text{in.}} \left[\frac{1\ \text{m}}{39.37\ \cancel{\text{in.}}}\right] = 0.0635\ \text{m}$$

$$(50\ \text{t})I = (280\ \text{At/m})(0.1397\ \text{m}) + (1500\ \text{At/m})(0.0635\ \text{m})$$
$$50I = 39.12 + 95.25 = 134.37$$
$$I = \mathbf{2.687\ A}$$

15. $$4\ \cancel{\text{cm}} \left[\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right] = 0.04\ \text{m}$$

$$f = \frac{1}{2} NI \frac{d\phi}{dx} = \frac{1}{2}(80\ \text{t})(0.9\ \text{A}) \frac{(8 \times 10^{-4}\ \text{Wb} - 0.5 \times 10^{-4}\ \text{Wb})}{\frac{1}{2}(0.04\ \text{m})} = \frac{36(7.5 \times 10^{-4})}{0.02}$$
$$= \mathbf{1.35\ N}$$

17. a. $$0.2\ \cancel{\text{cm}} \left[\frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right] = 2 \times 10^{-3}\ \text{m}$$

$$A = \frac{\pi d^2}{4} = \frac{(3.14)(0.01\ \text{m})^2}{4} = 0.785 \times 10^{-4}\ \text{m}^2$$
$$NI = H_g l_g,\ H_g = 7.96 \times 10^5\ B_g$$
$$(200\ \text{t})I = \left[(7.96 \times 10^5)\left[\frac{0.2 \times 10^{-4}\ \text{Wb}}{0.785 \times 10^{-4}\ \text{m}^2}\right]\right] 2 \times 10^{-3}\ \text{m}$$
$$I = \mathbf{2.028\ A}$$

b. $$F = \frac{1}{2}\frac{B_g^2 A}{\mu_o} = \frac{1}{2}\frac{(0.2548\ \text{T})^2(0.785 \times 10^{-4}\ \text{m}^2)}{4\pi \times 10^{-7}}$$
$$\cong \mathbf{2\ N}$$

19. $$NI = Hl$$
$$l = 2\pi r = (6.28)(0.08\ \text{m}) = 0.5024\ \text{m}$$
$$(100\ \text{t})(2\ \text{A}) = H(0.5024\ \text{m})$$
$$H = 398.09\ \text{At/m}$$
Fig. 11.24: $B \cong 0.675$ T
$$\Phi = BA = (0.675\ \text{T})(0.009\ \text{m}^2) = 0.0061\ \text{Wb}$$
$$\Phi = \mathbf{6.1 \times 10^{-3}\ Wb}$$

21. a. $1\tau = 0.632\ \text{T}_{max}$

$\text{T}_{max} \cong 1.5$ T for cast steel

$0.632(1.5\ \text{T}) = 0.945$ T

At 0.945 T, $H \cong 700$ At/m (Fig. 11.21)

$$\therefore\ \mathbf{B = 1.5(1 - e^{-H/700\ At/m})}$$

b. $H = 900$ At/m:

$$B = 1.5\left[1 - e^{-\frac{900\ \text{At/m}}{700\ \text{At/m}}}\right] = \mathbf{1.085\ T}$$

Graph: $\cong$ **1.1 T**

$H = 1800$ At/m:

$$B = 1.5\left[1 - e^{-\frac{1800\ \text{At/m}}{700\ \text{At/m}}}\right] = \mathbf{1.385\ T}$$

Graph: $\cong$ **1.38 T**
$H = 2700$ At/m:

$$B = 1.5\left[1 - e^{-\frac{2700\text{ At/m}}{700\text{ At/m}}}\right] = \mathbf{1.468\ T}$$

Graph: $\cong$ **1.47 T**
Excellent comparison!

c. $B = 1.5(1 - e^{-H/700\text{ At/m}}) = 1.5 - 1.5e^{-H/700\text{ At/m}}$

$B - 1.5 = -1.5e^{-H/700\text{ At/m}}$

$1.5 - B = 1.5e^{-H/700\text{ At/m}}$

$$\frac{1.5 - B}{1.5} = e^{-H/700\text{ At/m}}$$

$$\log_e\left[1 - \frac{B}{1.5}\right] = \frac{-H}{700\text{ At/m}}$$

and $H = -700 \log_e\left[1 - \dfrac{B}{1.5}\right]$

d. $B = 1$ T:

$$H = -700 \log_e\left[1 - \frac{1}{1.5}\right] = \mathbf{769.03\ At/m}$$

Graph: $\cong$ **750 At/m**

$B = 1.4$ T:

$$H = -700 \log_e\left[1 - \frac{1.4}{1.5}\right] = \mathbf{1895.64\ At/m}$$

Graph: $\cong$ **1920 At/m**

e. $$H = -700 \log_e\left[1 - \frac{B}{1.5}\right]$$
$$= -700 \log_e\left[1 - \frac{0.2}{1.5}\right]$$
$$= 100.2\text{ At/m}$$

$$I = \frac{Hl}{N} = \frac{(100.2\text{ At/m})(0.16\text{ m})}{400\text{ t}} = \mathbf{40.1\ mA}$$

vs 44 mA for Ex. 11.3

CHAPTER 11 (Even)

2. Φ: SI **6×10^{-4} Wb**, English **60,000 lines**
   B: SI **0.465 T**, CGS **4.65×10^3 Gauss**, English **30,000 lines/in.^{2}**

4. a. $\Re = \dfrac{l}{\mu A} = \dfrac{0.06 \text{ m}}{\mu 2 \times 10^{-4} \text{ m}^2} = \dfrac{300}{\mu\text{m}}$

   b. $\Re = \dfrac{l}{\mu A} = \dfrac{0.0762 \text{ m}}{\mu 5 \times 10^{-4} \text{ m}^2} = \dfrac{152.4}{\mu\text{m}}$

   c. $\Re = \dfrac{l}{\mu A} = \dfrac{0.1 \text{ m}}{\mu 1 \times 10^{-4} \text{ m}^2} = \dfrac{1000}{\mu\text{m}}$

   from the above $\Re_{(c)} > \Re_{(a)} > \Re_{(b)}$

6. $\Re = \dfrac{\mathscr{F}}{\Phi} = \dfrac{120 \text{ gilberts}}{72{,}000 \text{ maxwells}} = \mathbf{1.667 \times 10^{-3}}$ **rels (CGS)**

8. $\mu = \dfrac{2B}{H} = \dfrac{2(1200 \times 10^{-4} \text{ T})}{600 \text{ At/m}} = \mathbf{4 \times 10^{-4}}$ **Wb/Am**

10. $B = \dfrac{\Phi}{A} = \dfrac{3 \times 10^{-4} \text{ Wb}}{5 \times 10^{-4} \text{ m}^2} = 0.6 \text{ T}$

    Fig. 11.23, $H_{iron} = 2500$ At/m
    Fig. 11.24, $H_{steel} = 70$ At/m

    $$NI = Hl_{(iron)} + Hl_{(steel)}$$
    $$(100 \text{ t})I = (H_{iron} + H_{steel})l$$
    $$(100 \text{ t})I = (2500 \text{ At/m} + 70 \text{ At/m})0.3 \text{ m}$$
    $$I = \frac{771 \text{ A}}{100} = \mathbf{7.71 \text{ A}}$$

12. a. $80{,}000 \cancel{\text{lines}} \left[\dfrac{1 \text{ Wb}}{10^8 \cancel{\text{lines}}}\right] = 8 \times 10^4 \times 10^{-8} \text{ Wb} = 8 \times 10^{-4} \text{ Wb}$

    $$l_{(\text{cast steel})} = 5.5 \cancel{\text{in.}} \left[\frac{1 \text{ m}}{39.37 \cancel{\text{in.}}}\right] = 0.1397 \text{ m}$$

    $$l_{(\text{sheet steel})} = 0.5 \cancel{\text{in.}} \left[\frac{1 \text{ m}}{39.37 \cancel{\text{in.}}}\right] = 0.0127 \text{ m}$$

    $$\text{Area} = 1 \cancel{\text{in.}^2} \left[\frac{1 \text{ m}}{39.37 \cancel{\text{in.}}}\right]\left[\frac{1 \text{ m}}{39.37 \cancel{\text{in.}}}\right] = 6.45 \times 10^{-4} \text{ m}^2$$

    $$B = \frac{\Phi}{A} = \frac{8 \times 10^{-4} \text{ Wb}}{6.45 \times 10^{-4} \text{ m}^2} = 1.24 \text{ T}$$

Fig 11.24: $H_{\text{sheet steel}} \cong 460$ At/m, Fig. 11.23: $H_{\text{cast steel}} \cong 1275$ At/m

$$NI = Hl_{(\text{sheet steel})} + Hl_{(\text{cast iron})}$$
$$= (460 \text{ At/m})(0.0127 \text{ m}) + (1275 \text{ At/m})(0.1397 \text{ m})$$
$$= 5.842 \text{ At} + 178.12 \text{ At}$$
$$NI = \mathbf{183.96}$$

b. Cast steel: $\mu = \dfrac{B}{H} = \dfrac{1.24 \text{ T}}{1275 \text{ At/m}} = \mathbf{9.725 \times 10^{-4}\ Wb/Am}$

Sheet steel: $\mu = \dfrac{B}{H} = \dfrac{1.24 \text{ T}}{460 \text{ At/m}} = \mathbf{26.96 \times 10^{-4}\ Wb/Am}$

14. a. $l_{ab} = l_{ef} = 0.05$ m, $l_{af} = 0.02$ m, $l_{bc} = l_{de} = 0.0085$ m

$$NI = 2H_{ab}l_{ab} + 2H_{bc}l_{bc} + H_{fa}l_{fa} + H_g l_g$$

$$B = \frac{\Phi}{A} = \frac{2.4 \times 10^{-4} \text{ Wb}}{2 \times 10^{-4} \text{ m}^2} = 1.2 \text{ T} \Rightarrow H \cong 360 \text{ At/m} \quad \text{(Fig. 11.24)}$$

$$100I = 2(360 \text{ At/m})(0.05 \text{ m}) + 2(360 \text{ At/m})(0.0085 \text{ m})$$
$$+ (360 \text{ At/m})(0.02 \text{ m}) + 7.97 \times 10^5(1.2 \text{ T})(0.003 \text{ m})$$
$$= 36 \text{ At} + 6.12 \text{ At} + 7.2 \text{ At} + 2869 \text{ At}$$
$$100I = 2918.32 \text{ At}$$
$$I \cong \mathbf{29.18\ A}$$

b. air gap: metal = 2869 At:49.72 At = **58.17:1**

$$\mu_{\text{sheet steel}} = \frac{B}{H} = \frac{1.2 \text{ T}}{360 \text{ At/m}} = \mathbf{3.33 \times 10^{-3}\ Wb/Am}$$
$$\mu_{\text{air}} = \mathbf{4\pi \times 10^{-7}\ Wb/Am}$$
$$\mu_{\text{sheet steel}}:\mu_{\text{air}} = 3.33 \times 10^{-3} \text{ Wb/Am}:4\pi \times 10^{-7} \cong \mathbf{2627:1}$$

16. $C = 2\pi r = (6.28)(0.3 \text{ m}) = 1.884$ m

$$B = \frac{\Phi}{A} = \frac{2 \times 10^{-4} \text{ Wb}}{1.3 \times 10^{-4} \text{ m}^2} = 1.538 \text{ T}$$

Fig. 11.23: $H_{\text{sheet steel}} \cong 2100$ At/m

$$H_g = 7.97 \times 10^5 B_g = (7.97 \times 10^5)(1.538 \text{ T}) = 12.26 \times 10^5 \text{ At/m}$$
$$N_1 I_1 + N_2 I_2 = H_g l_g + Hl_{(\text{sheet steel})}$$
$$(200 \text{ t})I_1 + (40 \text{ t})(0.3 \text{ A}) = (12.26 \times 10^5 \text{ At/m})(2 \text{ mm}) + (2100 \text{ At/m})(1.884 \text{ m})$$
$$I_1 = \mathbf{31.98\ A}$$

18. **Table:**

Section	Φ(Wb)	A(m²)	B(T)	H	l(m)	Hl
a–b, g–h		5×10^{-4}			0.2	
b–c, f–g	2×10^{-4}	5×10^{-4}			0.1	
c–d, e–f	2×10^{-4}	5×10^{-4}			0.099	
a–h		5×10^{-4}			0.2	
b–g		2×10^{-4}			0.2	
d–e	2×10^{-4}	5×10^{-4}			0.002	

$$B_{bc} = B_{cd} = B_g = B_{ef} = B_{fg} = \frac{\Phi}{A} = \frac{2 \times 10^{-4}\ \text{Wb}}{5 \times 10^{-4}\ \text{m}^2} = 0.4\ \text{T}$$

Air gap: $H_g = 7.97 \times 10^5(0.4\ \text{T}) = 3.188 \times 10^5\ \text{At/m}$

$H_g l_g = (3.188 \times 10^5\ \text{At/m})(2\ \text{mm}) = 637.60\ \text{At}$

Fig 11.24: $H_{bc} = H_{cd} = H_{ef} = H_{fg} = 55\ \text{At/m}$

$H_{bc}l_{bc} = H_{fg}l_{fg} = (55\ \text{At/m})(0.1\ \text{m}) = 5.5\ \text{At}$

$H_{cd}l_{cd} = H_{ef}l_{ef} = (55\ \text{At/m})(0.099\ \text{m}) = 5.445\ \text{At}$

For loop 2: $\sum_{\circlearrowleft} \mathscr{F} = 0$

$H_{bc}l_{bc} + H_{cd}l_{cd} + H_g l_g + H_{ef}l_{ef} + H_{fg}l_{fg} - H_{gb}l_{gb} = 0$

$5.5\ \text{At} + 5.445\ \text{At} + 637.60\ \text{At} + 5.445\ \text{At} + 5.50\ \text{At} - H_{gb}l_{gb} = 0$

$H_{gb}l_{gb} = 659.49\ \text{At}$

and $H_{gb} = \dfrac{659.49\ \text{At}}{0.2\ \text{m}} = 3297.45\ \text{At/m}$

Fig 11.23: $B_{gb} \cong 1.55\ \text{T}$

with $\Phi_2 = B_{gb}A = (1.55\ \text{T})(2 \times 10^{-4}\ \text{m}^2) = 3.1 \times 10^{-4}\ \text{Wb}$

$\Phi_T = \Phi_1 + \Phi_2$

$= 2 \times 10^{-4}\ \text{Wb} + 3.1 \times 10^{-4}\ \text{Wb}$

$= 5.1 \times 10^{-4}\ \text{Wb} = \Phi_{ab} = \Phi_{ha} = \Phi_{gh}$

$$B_{ab} = B_{ha} = B_{gh} = \frac{\Phi_T}{A} = \frac{5.1 \times 10^{-4}\ \text{Wb}}{5 \times 10^{-4}\ \text{m}^2} = 1.02\ \text{T}$$

B–H curve: (Fig 11.24):

$H_{ab} = H_{ha} = H_{gh} \cong 180\ \text{At/m}$

$H_{ab}l_{ab} = (180\ \text{At/m})(0.2\ \text{m}) = 36\ \text{At}$

$H_{ha}l_{ha} = (180\ \text{At/m})(0.2\ \text{m}) = 36\ \text{At}$

$H_{gh}l_{gh} = (180\ \text{At/m})(0.2\ \text{m}) = 36\ \text{At}$

which **completes** the table!

Loop #1: $\sum_{\circlearrowleft} \mathscr{F} = 0$

$NI = H_{ab}l_{ab} + H_{bg}l_{bg} + H_{gh}l_{gh} + H_{ah}l_{ah}$

$(200\ \text{t})I = 36\ \text{At} + 659.49\ \text{At} + 36\ \text{At} + 36\ \text{At}$

$(200\ \text{t})I = 767.49\ \text{At}$

$I \cong \mathbf{3.84\ A}$

20. $NI = H_{ab}(l_{ab} + l_{bc} + l_{de} + l_{ef} + l_{fa}) + H_g l_g$

$300\ \text{At} = H_{ab}(0.7992\ \text{m}) + 7.97 \times 10^5\ B_g(0.8\ \text{mm})$

$300\ \text{At} = H_{ab}(0.7992\ \text{m}) + 637.6\ B_g$

Assuming $637.6\ B_g >> H_{ab}(0.7992\ \text{m})$

then $300\ \text{At} = 637.6\ B_g$

and $B_g = 0.471\ \text{T}$

$\Phi = BA = (0.471\ \text{T})(2 \times 10^{-4}\ \text{m}^2) = 0.942 \times 10^{-4}\ \text{Wb}$

$B_{ab} = B_g = 0.471\ \text{T} \Rightarrow H \cong 270\ \text{At/m}$ (Fig. 11.24)

$300\ \text{At} = (270\ \text{At/m})(0.7992\ \text{m}) + 637.6(0.471\ \text{T})$

$300\ \text{At} \neq 516.09\ \text{At}$

$\therefore$ Poor approximation!

$$\frac{300\text{ At}}{516.09\text{ At}} \times 100\% \cong 58\%$$

Reduce Φ to 58%

$$0.58(0.942 \times 10^{-4}\text{ Wb}) = 0.546 \times 10^{-4}\text{ Wb}$$

$$B = \frac{\Phi}{A} = \frac{0.546 \times 10^{-4}\text{ Wb}}{2 \times 10^{-4}\text{ m}^2} = 0.273\text{ T} \Rightarrow H \cong 190\text{ At/m} \quad (\text{Fig. 11.24})$$

$$300\text{ At} = (190\text{ At/m})(0.7992) + 637.6(0.273\text{ T})$$

$$300\text{ At} \neq 325.91$$

$$\text{Reduce } \Phi \text{ another } 10\% = 0.546 \times 10^{-4}\text{ Wb} - 0.1(546 \times 10^{-4}\text{ Wb})$$

$$= 0.491 \times 10^{-4}\text{ Wb}$$

$$B = \frac{\Phi}{A} = \frac{0.491 \times 10^{-4}\text{ Wb}}{2 \times 10^{-4}\text{ m}^2} = 0.246\text{ T} \Rightarrow H \cong 175\text{ At/m} \quad (\text{Fig. 11.24})$$

$$300\text{ At} = (175\text{ At/m})(0.7992) + 637.6(0.273\text{ T})$$

$300\text{ At} \neq 313.92\text{ At}$ but within 5% $\therefore$ OK

$\Phi \cong \mathbf{0.546 \times 10^{-4}}$ **Wb**